Received: *March 9, 2000*, Accepted: *April 6, 2000*

Superstring theory on $AdS_3 \times G/H$ and boundary $N = 3$ superconformal symmetry

Riccardo Argurio[a], Amit Giveon[ab] and Assaf Shomer[a]

[a] *Racah Institute of Physics, The Hebrew University Jerusalem 91904, Israel*
[b] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
E-mail: argurio@cc.huji.ac.il, shomer@cc.huji.ac.il, ygiveon@vms.huji.ac.il

Abstract: Superstrings propagating on backgrounds of the form $AdS_3 \times G/H$ are studied using the coset CFT approach. We focus on seven dimensional cosets which have a semiclassical limit, and which give rise to $N = 3$ superconformal symmetry in the dual CFT. This is realized for the two cases $AdS_3 \times \mathrm{SU}(3)/\,\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\,\mathrm{SO}(3)$, for which we present an explicit construction. We also provide sufficient conditions on a CFT background to enable a similar construction, and comment on the geometrical interpretation of our results.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua, Conformal Field Models in String Theory, Extended Supersymmetry.

JHEP04(2000)010

Contents

1. Introduction

String propagation on curved backgrounds with an AdS_3 factor has been of recent interest (see for instance [1]–[6], and [7] for additional references). One motivation is the fact that $AdS_3 \simeq \mathrm{SL}(2)$ is an exact background which can be treated in string pertubation theory, and thus allows to consider the AdS/CFT correspondence [8] beyond the supergravity limit. Some specific examples that were studied in this context include superstrings propagating on $AdS_3 \times \mathcal{N}$ where $\mathcal{N}$ was a group manifold [3, 9], or an orbifold of a group manifold [10, 11]. In this paper we study cases in which $\mathcal{N}$ is a coset manifold. This is an interesting generalization of the AdS/CFT correspondence which has been considered in the higher-dimensional cases of type-IIB string theory on $AdS_5 \times \mathcal{N}^5$ [12] and of M-theory on $AdS_4 \times \mathcal{N}^7$ [13, 14], where $\mathcal{N}^5$ and $\mathcal{N}^7$ are Einstein manifolds (generically coset manifolds) preserving a fraction of supersymmetry. This type of construction allows one to consider dual supersymmetric CFTs which are not "orbifolds" of the maximally supersymmetric one. The $AdS_3 \times \mathcal{N}$ case is somewhat different since here we have the possibility of studying $\mathcal{N}$ in the context of coset CFTs.

We choose to study coset CFTs which have a large radius (or large level k) semiclassical limit, corresponding to superstrings propagating on seven-dimensional coset manifolds. Moreover, we focus on cases in which the dual two-dimensional theory (also referred to as the spacetime CFT) has an extended superconformal symmetry.[1] Coset models leading to $N = 2$ can be easily realized as particular cases of the general construction of [15], where $\mathcal{N}$ decomposes as a U(1) factor times a Kazama-Suzuki model [16]. On the other hand, there are no seven-dimensional coset manifolds leading to $N = 4$ supersymmetry in spacetime (except of course the cases [3, 9] in which the cosets are actually group manifolds). Therefore, we shall be interested in the cases where the spacetime CFT has $N = 3$ supersymmetry.

Our main result, presented in sections 3 and 4, is the construction of the spacetime $N = 3$ superconformal algebra in the two cases:

$$AdS_3 \times \frac{\mathrm{SU}(3)}{\mathrm{U}(1)}\,, \qquad AdS_3 \times \frac{\mathrm{SO}(5)}{\mathrm{SO}(3)}\,, \tag{1.1}$$

which actually turn out to be the only coset models giving rise to $N = 3$. An interesting by-product of the construction is related to the fact that getting $N = 3$ depends on the choice of chiral GSO projection. Choosing the opposite projection leads to an $N = 1$ superconformal algebra in spacetime together with an SU(2) affine algebra acting trivially on the supercharges. This spacetime structure also appears in [11], where a $\mathbb{Z}_2$ orbifold of the large $N = 4$ algebra obtained by [9] is taken.

We also go beyond the coset setup by providing in section 6 a set of sufficient conditions on a CFT background $\mathcal{N}$ which allow for the construction of $N = 3$ superconformal symmetry in spacetime from superstrings on $AdS_3 \times \mathcal{N}$. We recover the $N = 1$ structure for the other GSO projection also in this general setup. The proof elaborates on the construction of $N = 2$ spacetime supersymmetry by [15], the additional ingredient being the presence in the CFT on $\mathcal{N}$ of an affine SU(2). Finally, we comment in section 7 on a possible geometrical interpretation of these conditions, and relate our work to the case of M-theory compactified on $AdS_4 \times \mathcal{N}^7$ [13, 14], as well as to brane configurations.

2. Spacetime $N = 3$ superconformal algebra

Extended superconformal algebras in two dimensions also include an affine R-symmetry algebra, which generally leads to a quantization of the central charge in unitary theories. Specifically, the $N = 3$ superconformal algebra has an affine SU(2) subalgebra. The central charge is related to the level $\tilde{k}$ of this affine SU(2), which is an integer, by $\tilde{c} = \frac{3}{2}\tilde{k}$ [17]. Therefore, a necessary condition for string theory on a background of the form $AdS_3 \times \mathcal{N}$ to have spacetime $N = 3$ superconformal symmetry is the existence of an affine SU(2) in spacetime. This is obtained when the worldsheet CFT on $\mathcal{N}$ has an affine SU(2) symmetry as well [3]. If the respective worldsheet levels of SL(2) and of SU(2) are k and k', the analysis of [3, 5] shows that

[1] In the following we refer to the supersymmetry of, say, the left-movers only. The supersymmetry of the other sector depends on the particular superstring theory considered.

in the spacetime theory we have $\tilde{c} = 6kp$ and $\tilde{k} = k'p$, where p is the integer number introduced in [3], related to the maximal number of "long strings" [5, 18]. A further condition is thus $k' = 4k$ (recall that k is not forced to be an integer).

In the following we focus on coset manifolds $\mathcal{N}$ which have 7 dimensions, so that a large k semi-classical limit is possible. Two cases which satisfy the conditions given above are $\mathrm{SL}(2)_k \times \mathcal{N}$ with:

$$\mathcal{N}_1 = \frac{\mathrm{SU}(3)_{4k}}{\mathrm{U}(1)} \tag{2.1}$$

and

$$\mathcal{N}_2 = \frac{\mathrm{SO}(5)_{4k}}{\mathrm{SO}(3)}\,. \tag{2.2}$$

Note that there are several ways of choosing the SO(3) in $\mathcal{N}_2$, according to the nesting of subgroups $\mathrm{SO}(3) \subset \mathrm{SO}(3) \times \mathrm{SO}(3) \simeq \mathrm{SO}(4) \subset \mathrm{SO}(5)$. Since we require $\mathcal{N}_2$ to have an unbroken SO(3) symmetry, we are forced to mod out by one of the two SO(3) factors of SO(4).[2] It is straightforward to show that the two models above are critical:

$$c_{sl} + c_1 = \left(\frac{9}{2} + \frac{6}{k}\right) + \left(12 - \frac{24}{4k} - \frac{3}{2}\right) = 15\,, \tag{2.3}$$

$$c_{sl} + c_2 = \left(\frac{9}{2} + \frac{6}{k}\right) + \left(15 - \frac{30}{4k} - \frac{9}{2} + \frac{6}{4k}\right) = 15\,. \tag{2.4}$$

We now show that these two models indeed possess $N = 3$ superconformal symmetry in spacetime by explicit construction. Since the construction is similar in the two cases, we will focus here on the first case, and then go briefly over the second one.

3. Superstring theory on $AdS_3 \times \mathrm{SU}(3)/\,\mathrm{U}(1)$

We first have to set some notations, starting from the SL(2) WZW part. We mainly follow the formalism of [16] and [3]. For simplicity we only treat the holomorphic sector.

The SL(2) supersymmetric WZW model is constituted of the three currents of the SL(2) affine algebra at level k, and the three fermions implied by the $N = 1$ worldsheet supersymmetry, satisfying the following OPEs:

$$\begin{aligned}
J^P(z)J^Q(w) &\sim \frac{(k/2)\eta^{PQ}}{(z-w)^2} + \frac{i\epsilon^{PQR}\eta_{RS}J^S(w)}{z-w}\,,\\
J^P(z)\psi^Q(w) &\sim \frac{i\epsilon^{PQR}\eta_{RS}\psi^S(w)}{z-w}\,,\\
\psi^P(z)\psi^Q(w) &\sim \frac{(k/2)\eta^{PQ}}{z-w}\,,
\end{aligned} \tag{3.1}$$

[2]Note that modding out by the diagonal SO(3) and then by a further U(1) would lead to a Kazama-Suzuki model [16].

where $P, Q, R, S = 1, 2, 3$, $\eta^{PQ} = (++-)$ and $\epsilon^{123} = 1$. As usual in supersymmetric WZW models, the currents can be decomposed in two pieces:

$$J^P = \hat{J}^P - \frac{i}{k}\eta^{PQ}\epsilon_{QRS}\,\psi^R\,\psi^S\,. \tag{3.2}$$

The first piece $\hat{J}^P$ constitutes an affine algebra at level $k+2$, and has regular OPE with the fermions ψ^P. We will thus refer to $\hat{J}^P$ as the bosonic currents. The second part constitutes an affine algebra at level -2, and is referred to as the fermionic part of the current.

The worldsheet stress-energy tensor and $N=1$ supercurrent are:

$$\begin{aligned} T_{sl} &= \frac{1}{k}\Big(\hat{J}^P\hat{J}_P - \psi^P\partial\psi_P\Big), \\ G_{sl} &= \frac{2}{k}\Big(\psi^P\hat{J}_P - \frac{i}{3k}\epsilon_{PQR}\psi^P\psi^Q\psi^R\Big)\,. \end{aligned} \tag{3.3}$$

Let us now turn to the SU(3)/ U(1) coset CFT. We start from the SU(3) affine superalgebra at level $k' = 4k$ realized as follows:

$$\begin{aligned} K^A(z)K^B(w) &\sim \frac{(k'/2)\delta^{AB}}{(z-w)^2} + \frac{if_{ABC}K^C(w)}{z-w}\,, \\ K^A(z)\chi^B(w) &\sim \frac{if_{ABC}\chi^C(w)}{z-w}\,, \\ \chi^A(z)\chi^B(w) &\sim \frac{(k'/2)\delta^{AB}}{z-w}\,. \end{aligned} \tag{3.4}$$

Here $A, B, C, D = 1, \ldots, 8$ and the structure constants f_{ABC} are $f_{123} = 1$, $f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = 1/2$ and $f_{458} = f_{678} = \sqrt{3}/2$. Since the metric is δ^{AB} we will not keep track of the upper or lower position of the SU(3) indices. As before, we split the currents into their bosonic and fermionic parts:

$$K^A = \hat{K}^A - \frac{i}{k'}f_{ABC}\,\chi^B\chi^C\,. \tag{3.5}$$

The bosonic currents realize an affine algebra at level $k'-3$.

We now choose to mod out the SU(3) by the U(1) generated by K^8. The SU(2) subgroup generated by K^1, K^2, K^3 is orthogonal to this U(1), and thus survives as an affine algebra in the coset CFT. The stress-energy tensor and the supercurrent of the coset CFT are built as in [16], using the decomposition $T_{\mathrm{SU(3)}} = T_{\mathrm{SU(3)/\,U(1)}} + T_{\mathrm{U(1)}}$, and similarly for the supercurrent G. The stress-energy tensor reads:

$$\begin{aligned} T_{\text{coset}} = {} & \frac{1}{k'}\Big(\hat{K}^1\hat{K}^1 + \cdots + \hat{K}^7\hat{K}^7\Big) - \frac{1}{k'}\Big(\chi^1\partial\chi^1 + \cdots + \chi^3\partial\chi^3\Big) - \\ & - \frac{1}{k'}\Big(1 - \frac{3}{2k'}\Big)(\chi^4\partial\chi^4 + \cdots + \chi^7\partial\chi^7) + \frac{2i\sqrt{3}}{k'^2}\hat{K}^8\Big(\chi^4\chi^5 + \chi^6\chi^7\Big) + \\ & + \frac{6}{k'^3}\chi^4\chi^5\chi^6\chi^7\,. \end{aligned} \tag{3.6}$$

Our goal now is to build the spacetime supercharges. For that we would like to construct spin-fields via bosonization following [19]. Note that since we are dealing with a coset and not with a group manifold, the fermions are generically not free. Of course since the SU(2) is preserved as an affine symmetry, the fermions belonging to it are free. Despite the above remark, we proceed to bosonize the 10 fermions into 5 bosons. This will actually uncover the interesting structure of the above coset model. Define:

$$\begin{aligned} \partial H_1 &= \frac{2}{k}\psi^1\psi^2\,, \qquad \partial H_2 = \frac{2}{k'}\chi^1\chi^2\,, \qquad i\partial H_3 = \frac{1}{k}\psi^3\chi^3\,, \\ \partial H_4 &= \frac{2}{k'}\chi^4\chi^5\,, \qquad \partial H_5 = \frac{2}{k'}\chi^6\chi^7\,. \end{aligned} \tag{3.7}$$

The scalars H_I are all canonically normalized: $H_I(z)H_J(w) \sim -\delta_{IJ}\log(z-w)$. Conversely, the fermions are given by:

$$\psi^1 = \frac{\sqrt{k}}{2}(e^{iH_1}+e^{-iH_1}), \qquad \psi^2 = \frac{i\sqrt{k}}{2}(e^{iH_1}-e^{-iH_1})\,, \tag{3.8}$$

and similarly for H_2, H_4 and H_5, while:

$$\psi^3 = \frac{\sqrt{k}}{2}(e^{iH_3}-e^{-iH_3}), \qquad \chi^3 = \frac{\sqrt{k'}}{2}(e^{iH_3}+e^{-iH_3})\,, \tag{3.9}$$

recalling that $H_3^\dagger = -H_3$ and $k' = 4k$.

In terms of these scalars, the total stress-energy tensor is:

$$\begin{aligned} T = T_{sl} + T_{\text{coset}} &= \frac{1}{k}(\hat{J}^1\hat{J}^1 + \hat{J}^2\hat{J}^2 - \hat{J}^3\hat{J}^3) + \frac{1}{k'}(\hat{K}^1\hat{K}^1 + \cdots + \hat{K}^7\hat{K}^7) - \\ &\quad - \frac{1}{2}(\partial H_1\partial H_1 + \partial H_2\partial H_2 + \partial H_3\partial H_3) + \frac{i\sqrt{3}}{k'}\hat{K}^8(\partial H_4 + \partial H_5) - \\ &\quad - \frac{1}{2}\Big(1 - \frac{3}{2k'}\Big)(\partial H_4\partial H_4 + \partial H_5\partial H_5) + \frac{3}{2k'}\partial H_4\partial H_5\,. \end{aligned} \tag{3.10}$$

Obviously, the scalars H_4 and H_5 are not free in the coset CFT. However, it is also easy to see that there is a linear combination of them which is free. This is what will enable us to build the $N=3$ spacetime superalgebra.

We thus write:

$$H_\pm = \frac{1}{\sqrt{2}}(H_4 \pm H_5)\,. \tag{3.11}$$

Our final expression for T is therefore:

$$\begin{aligned} T &= \frac{1}{k}(\hat{J}^1\hat{J}^1 + \hat{J}^2\hat{J}^2 - \hat{J}^3\hat{J}^3) + \frac{1}{k'}(\hat{K}^1\hat{K}^1 + \cdots + \hat{K}^7\hat{K}^7) - \\ &\quad - \frac{1}{2}(\partial H_1\partial H_1 + \partial H_2\partial H_2 + \partial H_3\partial H_3 + \partial H_-\partial H_-) - \\ &\quad - \frac{1}{2}\Big(1 - \frac{3}{k'}\Big)\partial H_+\partial H_+ + \frac{i\sqrt{6}}{k'}\hat{K}^8\partial H_+\,. \end{aligned} \tag{3.12}$$

We conclude that H_- is the fourth free scalar, namely that ∂H_- is a primary field of weight 1.

We now write the worldsheet $N=1$ supercurrent, which will be used to enforce the BRST condition on the spin fields. The supercurrent for the coset CFT reads:

$$G_{\text{coset}} = \frac{2}{k'}\Big(\chi^{\bar a}\hat K^{\bar a} - \frac{i}{3k'}f_{\bar a\bar b\bar c}\,\chi^{\bar a}\chi^{\bar b}\chi^{\bar c}\Big)\,, \tag{3.13}$$

where $\bar a$ are indices in the coset G/H. Putting together (3.3) and (3.13), substituting the structure constants of SU(3) and taking into account the bosonization in the term trilinear in the fermions, we get the expression for $G_{tot} = G_{sl} + G_{\text{coset}}$:

$$\begin{aligned} G_{tot} = {} & \frac{2}{k}(\psi^1\hat J^1 + \cdots - \psi^3\hat J^3) + \frac{2}{k'}(\chi^1\hat K^1 + \cdots + \chi^7\hat K^7) + \\ & + \frac{i}{\sqrt{k}}\Big\{\partial H_1(e^{iH_3} - e^{-iH_3}) - \frac{1}{2}\Big(\partial H_2 + \frac{1}{\sqrt{2}}\partial H_-\Big)(e^{iH_3} + e^{-iH_3})\Big\} + \\ & + \frac{1}{2\sqrt{k}}\Big(e^{iH_2 - i\sqrt{2}H_-} - e^{-iH_2 + i\sqrt{2}H_-}\Big)\,. \end{aligned} \tag{3.14}$$

Before going on to the BRST condition for the spin-fields, we write for completeness the expressions for the SU(2) currents, which remain primary fields of weight 1 in the coset CFT and can also be considered as the upper components of the fermions χ^1, χ^2, χ^3. Writing $K^\pm = K^1 \pm iK^2$ and similarly for the bosonic currents and the fermions, we get:

$$\begin{aligned} K^\pm &= \hat K^\pm \mp e^{\mp iH_2}(e^{iH_3} + e^{-iH_3}) \pm e^{\mp i\sqrt{2}H_-} \\ K^3 &= \hat K^3 - i\Big(\partial H_2 + \frac{1}{\sqrt{2}}\partial H_-\Big)\,. \end{aligned} \tag{3.15}$$

Note that since these currents are primaries of weight 1, this could have been an alternative way of showing that H_- is a free scalar.

3.1 Physical operators and the spacetime algebra

In order to construct the spacetime superconformal algebra we need, in particular, to construct physical supercharges which we choose to write in the $-1/2$ picture [19]:

$$Q \propto \oint e^{-\varphi/2}u^\alpha S_\alpha(z)\,dz\,. \tag{3.16}$$

Here S_α is a basis of spin-fields, u^α are constants, and φ is the bosonized superconformal ghost. The set of operators $e^{-\varphi/2}u^\alpha S_\alpha(z)$ should be BRST invariant and mutually local. We choose a basis of spin-fields

$$S_{[\epsilon_1\epsilon_2\epsilon_3\epsilon_-]} = e^{i/2\left(\epsilon_1 H_1 + \epsilon_2 H_2 + \epsilon_3 H_3 + \epsilon_- \sqrt{2}H_-\right)}\,, \tag{3.17}$$

where $\epsilon_I = \pm 1$. Because H_- is a free scalar, these 16 spin-fields are primaries of weight 5/8 and, therefore, $e^{-\varphi/2}u^\alpha S_\alpha(z)$ are primaries of weight 1, as they should be.

The super BRST condition on $e^{-\varphi/2}u^\alpha S_\alpha$ further requires that there will be no $(z-w)^{-3/2}$ singular terms in the OPE of $u^\alpha S_\alpha$ with the supercurrent G_{tot} (note that the only dangerous terms in G_{tot} are the ones trilinear in the fermions, i.e. the second and third lines in (3.14)). This leaves 8 combinations $u^\alpha S_\alpha$ out of the 16 spin-fields (3.17). The GSO condition, i.e. mutual locality, further leads to one of two choices of chirality: $\epsilon_1\epsilon_2\epsilon_3=-1$ or $\epsilon_1\epsilon_2\epsilon_3=1$, under which 6 or 2 of the combinations $u^\alpha S_\alpha$ survive, respectively.

Explicitly, the outcome of the computation is the following. For spacetime chirality $\epsilon_1\epsilon_2\epsilon_3=-1$, we get 6 physical spin-fields:

$$\begin{aligned}
S^+_{1/2} &= S_{[----]}\,,\\
S^-_{1/2} &= S_{[-+++]}\,,\\
S^3_{1/2} &= \frac{1}{2}(S_{[-++-]}-S_{[---+]})\,,\\
S^+_{-1/2} &= S_{[+-+-]}\,,\\
S^-_{-1/2} &= S_{[++-+]}\,,\\
S^3_{-1/2} &= \frac{1}{2}(S_{[++--]}-S_{[+-++]})\,.
\end{aligned} \tag{3.18}$$

The lower and upper labels of S^a_r denote respectively the quantum numbers of the global SL(2) and SU(2) symmetries, in the $(\mathbf{2},\mathbf{3})$ representation, as can be checked by taking the OPEs with the respective currents (3.2) and (3.15). For the other spacetime chirality $\epsilon_1\epsilon_2\epsilon_3=1$, we get 2 physical spin-fields:

$$\begin{aligned}
\tilde{S}_{1/2} &= \frac{1}{2}(S_{[--++]}+S_{[-+--]})\,,\\
\tilde{S}_{-1/2} &= \frac{1}{2}(S_{[+++-]}+S_{[+--+]})\,.
\end{aligned} \tag{3.19}$$

It can be checked that the above spin-fields $\tilde{S}_r$ have regular OPEs with the SU(2) currents.

We thus see that the choice of GSO projection will lead to different amounts of supersymmetry in spacetime. Namely, in a type II background, the projection in the left and right moving sectors of the worldsheet CFT determine, respectively, the amount of supersymmetry in the left and right moving sectors of the spacetime CFT. Specifically, the different GSO projections would lead in type IIA to $N=(3,1)$ or $N=(1,3)$, and in type IIB to $N=(3,3)$ or $N=(1,1)$. In the heterotic string, the different GSO projections in the worldsheet supersymmetric sector lead to $N=(3,0)$ or $N=(1,0)$ in spacetime.[3]

[3]Note that these examples provide, in particular, a construction of $N=1$ supersymmetry in

JHEP04(2000)010

The generators of the spacetime global $N = 3$ superconformal algebra are the following:

$$
\begin{aligned}
L_{\pm 1} &= -\oint dz J^{\pm}(z)\,, & L_0 &= -\oint dz J^3(z)\,, \\
T_0^{\pm} &= \oint dz K^{\pm}(z)\,, & T_0^3 &= \oint dz K^3(z)\,, \\
Q_{1/2}^{\pm} &= \oint dz e^{-\varphi/2} S_{1/2}^{\pm}(z)\,, & Q_{1/2}^3 &= \oint dz e^{-\varphi/2} S_{1/2}^3(z)\,, \\
Q_{-1/2}^{\pm} &= \oint dz e^{-\varphi/2} S_{-1/2}^{\pm}(z)\,, & Q_{-1/2}^3 &= \oint dz e^{-\varphi/2} S_{-1/2}^3(z)\,,
\end{aligned}
\tag{3.20}
$$

where we omit the normalization and the cocycle factors in the definition of the Q's. These operators close the global part of the $N = 3$ superconformal algebra (up to picture changing), in the NS sector:

$$
\begin{aligned}
[L_m, L_n] &= (m-n) L_{m+n}\,, \\
[T_0^a, T_0^b] &= i\epsilon^{abc} T_0^c\,, \\
[L_m, T_0^a] &= 0\,, \\
[L_m, Q_r^a] &= \Big(\frac{1}{2}m - r\Big) Q_{m+r}^a\,, \\
[T_0^a, Q_r^b] &= i\epsilon^{abc} Q_r^c\,, \\
\{Q_r^a, Q_s^b\} &= 2\delta^{ab} L_{r+s} + i\epsilon^{abc}(r-s) T_{r+s}^c\,,
\end{aligned}
\tag{3.21}
$$

where $m, n = 0, \pm 1,\ a, b, c = 1, 2, 3$ and $r, s = \pm 1/2$. Of course this model reproduces the full $N = 3$ superconformal algebra. The higher modes can be built as in [3, 5]. For instance, we can first construct all the L_n. Then acting with them on T_0^a and $Q_{\pm 1/2}^a$ one gets all the T_n^a and Q_r^a higher modes. To close the algebra an additional fermionic field is needed, all the modes of which are obtained from commutators of T_n^a and Q_r^a. The full algebra appears for example in [17, 11].

For completeness, we also write the (global) algebra for the other GSO projection, that is an $N = 1$ superconformal algebra together with an affine SU(2) which acts trivially on the supercharges. The supersymmetry generators are given in this case by:

$$
\tilde{Q}_{\pm 1/2} = \oint dz e^{-\varphi/2} \tilde{S}_{\pm 1/2}(z)\,,
\tag{3.22}
$$

spacetime which is not a $\mathbb{Z}_2$ orbifolding of the $N = 2$ construction of [15]. If it was such an orbifold, each of the supercharges would split into two BRST invariant pieces, leading to a total of 4 physical spin-fields, in contrast with the result (3.19).

and the algebra is:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n}\,,\\
[T_0^a, T_0^b] &= i\epsilon^{abc}T_0^c\,,\\
[L_m, T_0^a] &= 0\,,\\
[L_m, \tilde{Q}_r] &= \Big(\frac{1}{2}m - r\Big)\tilde{Q}_{m+r}\,,\\
[T_0^a, \tilde{Q}_r] &= 0\,,\\
\{\tilde{Q}_r, \tilde{Q}_s\} &= 2\delta^{ab}L_{r+s}\,.
\end{aligned}\tag{3.23}$$

Again, using the higher modes of L_n one can generate the higher modes of the other operators, together with the fermionic superpartners of the affine currents (see [11] for an analogous construction).

Let us conclude this section by a brief comment on a special case, when the level of the SU(3) is $k' = 3$ (the minimal level allowed by unitarity). We can decompose the coset CFT [SU(3)/ U(1)] into the product $[\mathrm{SU}(2)] \times [\mathrm{SU}(3)/(\mathrm{SU}(2) \times \mathrm{U}(1))]$. The central charge of the second piece is $c = 0$ in the $k' = 3$ case, thus the whole model reduces to string propagation on $\mathrm{SL}(2)_{3/4} \times \mathrm{SU}(2)_3$. If we consider the six dimensional model $\mathrm{SL}(2)_k \times \mathrm{SU}(2)_{k'}$, criticality enforces $1/k - 1/k' = 1$. The only combination which might allow $N = 3$ in spacetime (i.e. which verifies $k' = 4k$) is the one above, and our analysis indeed shows that it has $N = 3$. We will comment more on this case later.

4. Superstring theory on $AdS_3 \times \mathrm{SO}(5)/\,\mathrm{SO}(3)$

The construction of the $N = 3$ superconformal algebra in this case follows closely the steps of the previous section. We shall therefore be more schematical, and focus on the specifics of this model.

The SO(5) current algebra looks the same as (3.4), at the same level $k' = 4k$, but now the indices are $A, B, C, D = 1, \ldots, 9, 0$ and the structure constants f_{ABC} are $f_{123} = f_{456} = 1$, $f_{170} = f_{189} = -f_{279} = f_{280} = f_{378} = f_{390} = 1/2$ and $-f_{470} = f_{489} = -f_{579} = -f_{580} = f_{678} = -f_{690} = 1/2$. We work in the basis where the two orthogonal SO(3) subgroups of SO(5) are generated respectively by K^1, K^2, K^3 and K^4, K^5, K^6. We will mod out by the second one, leaving the first one as the R-symmetry.

As before, we straightforwardly bosonize the 10 fermions in the SL(2) and in the coset, to get:

$$\begin{aligned}
&\partial H_1 = \frac{2}{k}\psi^1\psi^2, \qquad \partial H_2 = \frac{2}{k'}\chi^1\chi^2, \qquad i\partial H_3 = \frac{1}{k}\psi^3\chi^3,\\
&\partial H_4 = \frac{2}{k'}\chi^7\chi^8, \qquad \partial H_5 = \frac{2}{k'}\chi^9\chi^0\,.
\end{aligned}\tag{4.1}$$

We can now write the total stress-energy tensor in terms of them as (we also use (3.11)):

$$
\begin{aligned}
T = {} & \frac{1}{k}\Big(\hat{J}^1\hat{J}^1 + \hat{J}^2\hat{J}^2 - \hat{J}^3\hat{J}^3\Big) + \frac{1}{k'}\Big(\hat{K}^1\hat{K}^1 + \cdots + \hat{K}^3\hat{K}^3 + \hat{K}^7\hat{K}^7 + \cdots + \hat{K}^0\hat{K}^0\Big) - \\
& - \frac{1}{2}(\partial H_1\partial H_1 + \partial H_2\partial H_2 + \partial H_3\partial H_3 + \partial H_+\partial H_+) - \frac{1}{2}\Big(1 - \frac{3}{k'}\Big)\partial H_-\partial H_- + \\
& + \frac{i\sqrt{2}}{k'}\hat{K}^6\partial H_- - \frac{1}{k'}\hat{K}^4\Big(e^{i\sqrt{2}H_-} - e^{-i\sqrt{2}H_-}\Big) - \frac{i}{k'}\hat{K}^5\Big(e^{i\sqrt{2}H_-} + e^{-i\sqrt{2}H_-}\Big) . \qquad (4.2)
\end{aligned}
$$

Now H_+ is the free scalar.

The analogous expression for the total supercurrent is:

$$
\begin{aligned}
G_{tot} = {} & \frac{2}{k}\Big(\psi^1\hat{J}^1 + \cdots - \psi^3\hat{J}^3\Big) + \frac{2}{k'}\Big(\chi^1\hat{K}^1 + \cdots + \chi^3\hat{K}^3 + \chi^7\hat{K}^7 + \ldots + \chi^0\hat{K}^0\Big) + \\
& + \frac{i}{\sqrt{k}}\Big\{\partial H_1\Big(e^{iH_3} - e^{-iH_3}\Big) - \frac{1}{2}\Big(\partial H_2 + \frac{1}{\sqrt{2}}\partial H_+\Big)\Big(e^{iH_3} + e^{-iH_3}\Big)\Big\} - \\
& - \frac{1}{2\sqrt{k}}\Big(e^{iH_2 - i\sqrt{2}H_+} - e^{-iH_2 + i\sqrt{2}H_+}\Big) . \qquad (4.3)
\end{aligned}
$$

The SU(2) currents are:

$$
\begin{aligned}
K^{\pm} &= \hat{K}^{\pm} \mp e^{\mp iH_2}(e^{iH_3} + e^{-iH_3}) \mp e^{\mp i\sqrt{2}H_+} \\
K^3 &= \hat{K}^3 - i\Big(\partial H_2 + \frac{1}{\sqrt{2}}\partial H_+\Big) . \qquad (4.4)
\end{aligned}
$$

The solutions to the BRST invariance conditions on the spin-fields $S_{[\epsilon_1\epsilon_2\epsilon_3\epsilon_+]}$ are, for the $\epsilon_1\epsilon_2\epsilon_3 = -1$ GSO projection:

$$
\begin{aligned}
S^+_{1/2} &= S_{[----]} , \\
S^-_{1/2} &= S_{[-+++]} , \\
S^3_{1/2} &= \frac{1}{2}(S_{[-++-]} + S_{[---+]}) , \\
S^+_{-1/2} &= S_{[+-+-]} , \\
S^-_{-1/2} &= S_{[++-+]} , \\
S^3_{-1/2} &= \frac{1}{2}(S_{[++--]} + S_{[+-++]}) . \qquad (4.5)
\end{aligned}
$$

For the other chirality, $\epsilon_1\epsilon_2\epsilon_3 = 1$, we get:

$$
\begin{aligned}
\tilde{S}_{1/2} &= \frac{1}{2}(S_{[--++]} - S_{[-+--]}) , \\
\tilde{S}_{-1/2} &= \frac{1}{2}(S_{[+++-]} - S_{[+--+]}) . \qquad (4.6)
\end{aligned}
$$

From the above spin-fields and currents, the construction of the $N = 3$ (or $N = 1$ according to the GSO projection) algebra proceeds in exactly the same manner as in the former case.

5. $N = 3$ superalgebra as an enhancement of $N = 2$

Since the $N = 3$ superconformal algebra has the $N = 2$ superalgebra as a subalgebra, and since general conditions for the appearance of the latter are known [15], it is natural and instructive to investigate the relation between the two constructions.

In [15] it was found that a general condition for having $N = 2$ superconformal algebra in spacetime for a background of the form $AdS_3 \times \mathcal{N}$, is the existence of an affine U(1) current in $\mathcal{N}$, such that $\mathcal{N}/\,\mathrm{U}(1)$ has $N = 2$ worldsheet supersymmetry. It was noted there that the U(1) must be chosen carefully in the cases where enhancement to $N > 2$ is expected, in order to embed the $N = 2$ construction into the explicit construction of the larger algebra.

We now proceed to show that in our two cases there is only one choice of the complex structure in $\mathcal{N}/\,\mathrm{U}(1)$, where the U(1) is the Cartan subalgebra of the SU(2), that leads to an $N = 2$ construction which is a subalgebra of the $N = 3$ algebra constructed in the previous sections.

The general construction of the $N = 2$ superconformal algebra for coset models [16] leads to the following U(1) R-current:

$$J_R = \frac{i}{k'} h_{\bar{a}\bar{b}}\, \chi^{\bar{a}} \chi^{\bar{b}} + \frac{1}{k'} h_{\bar{a}\bar{b}}\, f_{\bar{a}\bar{b}C} \left(\hat{K}^C - \frac{i}{k'} f_{C\bar{d}\bar{e}} \chi^{\bar{d}} \chi^{\bar{e}} \right). \tag{5.1}$$

The index C can run over both H and G/H. The complex structure $h_{\bar{a}\bar{b}}$ has to satisfy conditions that can be found in [16].

The construction of $N = 2$ supercharges then proceeds as follows [15].[4] We present a canonically normalized scalar H_0:

$$i\sqrt{3}\partial H_0 = J_R - \frac{4}{k'} K^3 , \tag{5.2}$$

which is used to construct the spin-fields:

$$S = e^{i/2}(\epsilon_1 H_1 + \epsilon_3 H_3 + \epsilon_0 \sqrt{3} H_0) , \tag{5.3}$$

where H_1 and H_3 are built as before (3.7). The BRST condition will pick up 4 of the above spin-fields as physical.[5] This construction of $N = 2$ will be embedded in our $N = 3$ constructions provided that the above spin-fields (5.3) can be rewritten as special cases of (3.17).

Let us consider first the $\mathrm{SU}(3)/\,\mathrm{U}(1)$ case. Here we have to look for complex structures of the six dimensional manifold $\mathrm{SU}(3)/\,\mathrm{U}(1)^2$. We take the Cartan subalgebra of the SU(3) to be generated by K^3 and K^8. We find three possible complex structures.

[4] Note that this is not the standard construction [20] with respect to $N = 2$ supersymmetry on the worldsheet.

[5] Note that the BRST condition in this construction leads to a definite GSO projection (i.e. no BRST invariant spin-fields of the other chirality are explicitly constructed).

The first complex structure is given by $h_{12} = h_{45} = h_{67} = 1$, and leads to an R-current of the form:

$$J'_R = i\partial H_2 + i\sqrt{2}\partial H_+ + \frac{2}{k'}K^3 + \frac{2\sqrt{3}}{k'}K^8 \,. \tag{5.4}$$

Note that the above expression is such that both ∂H_+ and K^8 will appear in the definition (5.2) of H_0, and therefore the spin-fields (5.3) cannot be matched to (3.17).

The second complex structure is given by $h_{12} = -h_{45} = -h_{67} = 1$, and leads to an R-current of the form:

$$J''_R = i\,\partial H_2 - i\sqrt{2}\,\partial H_+ + \frac{2}{k'}K^3 - \frac{2\sqrt{3}}{k'}K^8 \,. \tag{5.5}$$

The same remark as above applies here. Moreover these two currents are the sum and difference of the $N = 2$ U(1) R-currents that one can get by decomposing this coset CFT into $[\mathrm{SU}(2)/\,\mathrm{U}(1)] \times [\mathrm{SU}(3)/(\mathrm{SU}(2) \times \mathrm{U}(1))]$. We will explain shortly why this direct-product decomposition cannot lead to the enhancement to $N = 3$.

The third complex structure is given by $h_{12} = h_{45} = -h_{67} = 1$, and leads to an R-current of the form:

$$J_R = i\,\partial H_2 + i\sqrt{2}\,\partial H_- + \frac{4}{k'}K^3 \,. \tag{5.6}$$

The boson constructed as in (5.2) now reads $\sqrt{3}H_0 = H_2 + \sqrt{2}H_-$. The spin-fields (5.3) are thus exactly of the form (3.17) with $\epsilon_2 = \epsilon_-$. The BRST invariant ones are exactly the $N = 2$ subalgebra generators of (3.18), namely $S^{\pm}_{\pm 1/2}$.

Moving to the $\mathrm{SO}(5)/\,\mathrm{SO}(3)$ case, we have to consider the complex structure of $\mathrm{SO}(5)/(\mathrm{SO}(3) \times \mathrm{SO}(2))$, where the SO(2) is again the Cartan subalgebra of the remaining SO(3), generated by K^3. This case is different, as there is only one possible complex structure, given by $h_{12} = h_{78} = h_{90} = 1$. The associated R-current is:

$$J_R = i\,\partial H_2 + i\,\sqrt{2}\partial H_+ + \frac{4}{k'}K^3 \,. \tag{5.7}$$

As for (5.6), the BRST invariant spin-fields constructed according to [15] are the $N = 2$ supercharges $S^{\pm}_{\pm 1/2}$ of (4.5). The presence of only one complex structure in this case (as opposed to three in the previous one) is due to the fact that the four dimensional coset $\mathrm{SO}(5)/\,\mathrm{SO}(4)$ has no complex structure.

6. General conditions for obtaining $N = 3$

The above discussion leads us to present general conditions for the appearance of the $N = 3$ superconformal algebra in the context of string theory on $AdS_3 \times \mathcal{N}$. Such a background leads to $N = 3$ superconformal algebra in spacetime provided that:

(i) $\mathcal{N}$ has an affine SU(2) current algebra at level $k' = 4k$, where k is the level of SL(2).

(ii) $\mathcal{N}/\,\mathrm{U}(1)$ has $N = 2$ worldsheet supersymmetry, where U(1) is the Cartan subalgebra of the above SU(2). This condition alone allows one to construct an $N = 2$ superconformal algebra in spacetime (for a definite GSO projection).

(iii) This spacetime $N = 2$ algebra is enhanced to $N = 3$ if the scalar H_0 constructed as in (5.2) can be decomposed as $\sqrt{3}H_0 = H_2 + \sqrt{2}\tilde{H}_0$, where H_2 derives from the bosonization of the two remaining charged fermions of the SU(2), and $\tilde{H}_0$ is orthogonal to it.

Interestingly, these conditions imply as a by-product that for the opposite GSO projection we also get supersymmetry in spacetime, namely $N = 1$.

Let us present the proof by constructing the $N = 3$ superalgebra generators given the above conditions. Recall that besides the scalar (5.2), we also define [15] the scalars $\partial H_1 = (2/k)\psi^1\psi^2$ and $i\partial H_3 = (1/k)\psi^3\chi^3$. The existence of the affine SU(2), of which χ^3 is the lower component of the Cartan generator, allows us to define also $\partial H_2 = (2/k')\chi^1\chi^2$. Consider now the currents K^3 and $K^\pm$. Since they form an SU(2) supersymmetric WZW model (embedded inside the CFT on $\mathcal{N}$), they can be split into orthogonal pieces:

$$K^3 = \tilde{K}^3 - i\,\partial H_2, \qquad K^\pm = \tilde{K}^\pm \mp \frac{2}{\sqrt{k'}}\, e^{\mp i H_2}\chi^3\,. \tag{6.1}$$

We start now by noting that condition (iii) implies the following (making use of (5.2)):

$$i\sqrt{2}\,\partial\tilde{H}_0(z)K^3(w) \sim -\frac{1}{(z-w)^2}\,. \tag{6.2}$$

This means that K^3 can be split further:

$$K^3 = \hat{K}^3 - \frac{i}{\sqrt{2}}\partial\tilde{H}_0 - i\partial H_2\,, \tag{6.3}$$

where $\hat{K}^3$ has a regular OPE with $\tilde{H}_0$ (and of course H_2). Similarly, the currents $K^\pm$ also split into a "bosonic" part $\hat{K}^\pm$ which realizes an affine $\mathrm{SU}(2)_{k'-3}$, an $\mathrm{SU}(2)_1$ part built from $\tilde{H}_0$ and the usual fermionic $\mathrm{SU}(2)_2$ piece:

$$K^\pm = \hat{K}^\pm \mp e^{\mp i\sqrt{2}\tilde{H}_0} \mp e^{\mp iH_2}(e^{iH_3} + e^{-iH_3})\,. \tag{6.4}$$

We can now construct the 4 physical spin-fields as in [15]. Note that the presence of the full SU(2) is irrelevant in this step. Using the spin-fields of the form (5.3), we get $S_{[\epsilon_1\epsilon_3\epsilon_0]} = S_{[---]},\ S_{[-++]},\ S_{[++-]},\ S_{[+-+]}$. Splitting $\sqrt{3}H_0 = H_2 + \sqrt{2}\tilde{H}_0$, we can rewrite them as:

$$S_{[\epsilon_1\epsilon_2\epsilon_3\tilde{\epsilon}_0]} = S_{[----]},\ S_{[-+++]},\ S_{[+-+-]},\ S_{[++-+]}\,. \tag{6.5}$$

We now find the additional two BRST invariant supercharges, by acting on the above spin-fields with the SU(2) ladder operators $K^{\pm}$, which are also BRST invariant (as upper components of primaries of weight 1/2). The result is:

$$\frac{1}{2}(S_{[---+]}+S_{[-++-]}),\qquad \frac{1}{2}(S_{[+-++]}+S_{[++--]})\,. \tag{6.6}$$

The set of spin-fields (6.5)– (6.6) matches exactly the ones found in the cases detailed in the previous sections (note that the apparent sign difference with (3.18) and (3.15) can be absorbed in a redefinition of fields; in section 3 we preferred to stick to the usual Gell-Mann basis of SU(3)).

Defining the spacetime operators as in (3.20), one can show that the $N=3$ superconformal algebra closes.

The above proof builds upon the existence of the $N=2$ superalgebra, enhancing it to $N=3$ using the SU(2) currents. An alternative way of building the $N=3$ superalgebra, which also reveals the existence of the $N=1$ superalgebra for the other GSO projection, is to decompose the supercurrent of the CFT on $\mathcal{N}$ into an SU(2) part and a $\mathcal{N}/\,\mathrm{SU}(2)$ one. It can then be used to directly find all of the 8 physical spin-fields, 6 of one chirality and 2 of the other. The SU(2) part of the supercurrent is:

$$G_{\mathrm{SU}(2)}=\frac{2}{k'}\Big(\frac{1}{2}\chi^+\tilde{K}^-+\frac{1}{2}\chi^-\tilde{K}^++\chi^3\tilde{K}^3-\frac{2i}{k'}\chi^1\chi^2\chi^3\Big)\,. \tag{6.7}$$

Using (6.1), (6.3) and (6.4), and the bosonization, the relevant part of $G_{\mathrm{SU}(2)}$ for the BRST condition (i.e. the one that might lead to $(z-w)^{-3/2}$ singular terms in the OPE with the spin-fields) is:

$$\begin{aligned}G_{\mathrm{SU}(2)}=\cdots+\frac{1}{\sqrt{k'}}\Big\{-i\Big(\partial H_2+\frac{1}{\sqrt{2}}\partial\tilde{H}_0\Big)(e^{iH_3}+e^{-iH_3})\,-\\ -\Big(e^{iH_2-i\sqrt{2}\tilde{H}_0}-e^{-iH_2+i\sqrt{2}\tilde{H}_0}\Big)\Big\}\,.\end{aligned} \tag{6.8}$$

The first piece will give rise to a $(z-w)^{-3/2}$ singularity only when $\epsilon_2=\tilde{\epsilon}_0$, while the second piece will do so only when $\epsilon_2=-\tilde{\epsilon}_0$. Choosing $\epsilon_2=\tilde{\epsilon}_0$, we get 4 physical spin-fields of the same chirality. For $\epsilon_2=-\tilde{\epsilon}_0$ we get 4 physical spin-fields, two of each chirality.

It is worth noting that the direct product $\mathcal{N}=\mathrm{SU}(2)_{k'}\times\mathcal{N}'$, which manifestly fulfills conditions (i) and (ii), does not fulfill the third condition (except for one case which will be discussed shortly). The reason for this is the following. If H_0 fulfills condition (iii), it is straightforward to compute the OPE of $i\partial H_2$ with J_R, the R-current of $\mathcal{N}/\,\mathrm{U}(1)$, the result being:

$$i\partial H_2(z)J_R(w)\sim\frac{1-4/k'}{(z-w)^2}\,. \tag{6.9}$$

However, if $\mathcal{N} = \mathrm{SU}(2)_{4k} \times \mathcal{N}'$, then:

$$J_R = J_R^{\mathrm{SU}(2)/\,\mathrm{U}(1)} + J'_R = i\partial H_2 + \frac{2}{k'} K^3 + J'_R\,. \tag{6.10}$$

Since $i\partial H_2$ has a regular OPE with J'_R, its OPE with J_R in (6.10) gives a double pole with a residue of $(1 - 2/k')$ instead of (6.9). Thus such a CFT with a direct product SU(2) factor will not lead to $N = 3$ in spacetime through the mechanism described above.

This seems to contradict what was noted about the limiting $k' = 3$ case, which was reduced to $\mathrm{SL}(2)_{3/4} \times \mathrm{SU}(2)_3$. This is resolved by noting that we can always take J_R to $-J_R$ (this amounts to changing an overall sign in the complex structure, see (5.1)). Doing this, the OPE of $i\partial H_2$ with J_R gives a residue of $-(1-2/k')$, which is equal to $(1 - 4/k')$ only if $k' = 3$, that is in this particular case. For completeness, we sketch the construction in this case. Using (5.2) and (6.10), we write:

$$i\sqrt{3}\partial H_0 = -J_R - \frac{4}{3}K^3 = -i\partial H_2 - 2K^3 = i\partial H_2 - 2\tilde{K}^3\,. \tag{6.11}$$

Now we use the fact that the bosonic SU(2) WZW model at level 1 can be reformulated as the CFT of a free scalar at its self-dual radius. Denoting this scalar by $\tilde{H}_0$, we have $\tilde{K}^3 = -i/\sqrt{2}\partial\tilde{H}_0$ and $\tilde{K}^\pm = e^{\mp i\sqrt{2}\tilde{H}_0}$. It is thus clear that H_0 fulfills condition (iii), and the construction of the $N = 3$ superalgebra then proceeds as before.

7. Comments

Let us first comment on the relation between the general construction of the previous section and the two specific coset CFTs discussed before. It is possible to check that the only 7-dimensional cosets $\mathcal{N}$ which have an SU(2) symmetry (and do not factorize into a direct product $\mathrm{SU}(2) \times \mathcal{N}'$) are precisely $\mathcal{N}_1$ and $\mathcal{N}_2$ of eqs. (2.1) and (2.2). It is interesting that condition (i) of the previous section together with the requirement of having a semiclassical $k \to \infty$ limit automatically lead to models which fulfill the two remaining conditions. Considering 7 dimensional group manifolds, only the case $\mathcal{N} = \mathrm{SU}(2)_{2k} \times \mathrm{SU}(2)_{2k} \times \mathrm{U}(1)$ satisfies condition (i), where the $\mathrm{SU}(2)_{4k}$ is the diagonal one. It is straightforward to show that this manifold also satisfies conditions (ii) and (iii). This manifold actually possesses large $N = 4$ superconformal symmetry [9], of which $N = 3$ is a subalgebra. This fact was used in [11] to break $N = 4$ to $N = 3$ through a $\mathbb{Z}_2$ orbifold construction. It would be interesting to find the relation between this small set of models which share the same spacetime superconformal symmetry.

We conclude by commenting on the geometrical interpretation. It would be nice to translate the conditions we impose on the CFT on $\mathcal{N}$ into conditions on the geometry of the manifold. It is clear as was commented before that the $S^3 \cong \mathrm{SU}(2)$

has to be non-trivially fibered over the 4 dimensional base $\mathcal{N}/\,\mathrm{SU}(2)$. A related, but different, problem was actually discussed in the literature [13, 14], where conditions on Einstein 7-manifolds $\mathcal{N}$ are found in order to get different amounts of supersymmetry when considering 11 dimensional supergravity on $AdS_4 \times \mathcal{N}$. The condition for getting $N = 3$ in AdS_4 is that $\mathcal{N}$ has a tri-sasakian structure (in other words, the cone over it $C(\mathcal{N})$ is hyperKähler; see for instance [14] for the notions introduced hereafter). The above geometries are considered as near horizon geometries of M2-branes at the singularity of the Ricci-flat cone $C(\mathcal{N})$ over such manifolds. The tri-sasakian structure implies the presence of 3 Killing vectors forming an SO(3) algebra which rotates the 3 Killing spinors. It turns out that the only 7-dimensional tri-sasakian manifolds (satisfying some additional regularity conditions, see also [21]) are exactly the cosets[6] $\mathrm{SU}(3)/\,\mathrm{U}(1)$ and $\mathrm{SO}(5)/\,\mathrm{SO}(3) \cong S^7$. In the second case, the quotient is taken as in section 4 [22]. Note also that S^7 is trivially tri-sasakian since it has $N = 8$ Killing spinors.

Tri-sasakian manifolds can be seen as S^3 fibrations over a base, in the above cases $\mathbb{CP}^2$ and S^4 respectively. It is interesting to note that "squashing" a tri-sasakian manifold (i.e. rescaling the fiber with respect to the base) leads, for a definite value of the squashing parameter, to another Einstein manifold having one Killing spinor (instead of 3) and an unbroken SO(3) algebra of Killing vectors which acts trivially on the spinor. This is reminiscent of our results, where the different amounts of supersymmetry, $N = 3$ or $N = 1$, depend however on the GSO projection.

We should nevertheless stress that in spite of these similarities, there are a few differences. For instance, superstring theory on $AdS_3 \times \mathcal{N}$ does not require $\mathcal{N}$ to be an Einstein manifold.[7] Recall that the metric of the coset CFT sigma model, which can be obtained by gauging the WZW model on the group G and integrating out the gauge fields, is not the same as the metric on the homogeneous G/H coset space. Thus presumably the direct relation between the two issues is more algebraic in nature than geometrical.

Another question regards the brane configuration which might lead to the models considered here in the near horizon limit. Since we are dealing with pure NSNS backgrounds in type II theories, we expect such a brane configuration to involve fundamental strings and NS5-branes intersecting on the string, and possibly at non-trivial angles. Indeed it is known that the $AdS_3 \times S^3 \times T^4$ and the $AdS_3 \times S^3 \times S^3 \times S^1$ backgrounds are the near horizon geometries of configurations involving respectively a fundamental string within a NS5-brane [3], and an additional NS5-brane inter-

[6] The case $S^3 \times S^3 \times S^1$, which has $N = 3$ as a subgroup of large $N = 4$, does not appear in the classification above since its metric cannot be rescaled to become an Einstein manifold because of the flat S^1 factor.

[7] This might explain why in the $\mathrm{SO}(5)/\,\mathrm{SO}(3)$ case discussed in section 4 we do not find $N = 8$ supersymmetry but only $N = 3$.

secting the other orthogonally on the string [23, 24]. The latter brane configuration can be generalized by introducing a non-trivial angle between the two NS5-branes, but still requiring that some supersymmetries are preserved. This problem has been studied from the supergravity solution point of view in [25, 26], and it can be reduced to a problem of classifying the holonomy of an 8-dimensional manifold (the manifold which is orthogonal to the string intersection). It turns out [25] that solutions preserving a fraction of 3/16 of the supersymmetries are associated with 8-dimensional manifolds of holonomy $\mathrm{Sp}(4) \cong \mathrm{U}(2, \mathbf{H}) \cong \mathrm{SO}(5) \subset \mathrm{SO}(8)$, which are thus hyperKähler. This configuration would lead to $N = (3, 3)$ supersymmetry in type IIA.[8] This is still different from what we are looking for. In [26] more general NS5-brane configurations are considered, which are related to hyperKähler manifolds with torsion. The torsion allows for the holonomy to be different for the two chiralities of the spinors (again in type IIA). In this set up, solutions which have $N = (3, 1)$ supersymmetry are found, the associated manifold having Sp(4) holonomy for one chirality and Spin(7) holonomy for the other. It should be noted that the latter solutions, as presented in [26], do not include fundamental strings. The near horizon geometry of some of the configurations above is considered in [27], where it is found to be actually $AdS_3 \times S^3 \times S^3 \times S^1$ like in the orthogonal case (hinting towards a large $N = 4$ dual CFT, instead of an $N = 3$ one).

It is not straightforward to see how our coset manifolds could arise as near horizon geometries, at least in this context. In order to investigate this problem, one might need to further characterize the 8-dimensional hyperKähler manifolds involved in the brane configurations. Note that the latter are asymptotically flat, and thus generically not in the same class as the conical ones discussed previously, which were related to the classification of tri-sasakian (coset) manifolds.

An alternative to configurations with branes at angles, is to consider the near horizon limit of NS5-branes wrapping on 4-cycles, together with fundamental strings stretched along the unwrapped direction. These are expected [28] to be equivalent to a superstring on $AdS_3 \times \mathcal{N}$ where $\mathcal{N}$ has an affine U(1) symmetry and $\mathcal{N}/\mathrm{U}(1)$ is an $N = 2$ SCFT related to the geometry of the 4-cycle. For particular geometries, the $\mathcal{N}/\mathrm{U}(1)$ SCFT was identified with the infrared limit of $N = 2$ Landau-Ginzburg (LG) models [28]. In the examples considered in this work $\mathcal{N}/\mathrm{U}(1)$ is $\mathrm{SU}(3)/\mathrm{U}(1)^2$ and $\mathrm{SO}(5)/(\mathrm{SO}(3) \times \mathrm{U}(1))$. Generically, these $N = 2$ quotients do not have a LG description (except for the lowest levels of the $\mathrm{SU}(3)/\mathrm{U}(1)^2$ case). Therefore, the relation to brane configurations along these lines requires the understanding of the duality of [28] in models of the form $AdS_3 \times G/H$, where $G/(H \times \mathrm{U}(1))$ is a generic $N = 2$ quotient.

[8] The supersymmetry of the full brane configuration in type IIB is further reduced. For instance, the orthogonal intersection already preserves only 1/8 of the supersymmetries.

Acknowledgments

We are happy to thank S. Elitzur, O. Feinerman, D. Kutasov, W. Lerche, E. Rabinovici, M. Roček and A. Tomasiello for discussions and helpful remarks. A. G. also thanks the ITP in Santa Barbara for hospitality during the initial stages of this work. This work is supported in part by the Israel Academy of Sciences and Humanities — Centers of Excellence Program, and by the BSF — American-Israel Bi-National Science Foundation.

References

[1] J. Maldacena and A. Strominger, *AdS_3 black holes and a stringy exclusion principle*, *J. High Energy Phys.* **12** (1998) 005 [hep-th/9804085].

[2] J.M. Evans, M.R. Gaberdiel and M.J. Perry, *The no-ghost theorem for AdS_3 and the stringy exclusion principle*, *Nucl. Phys.* **B 535** (1998) 152 [hep-th/9806024].

[3] A. Giveon, D. Kutasov and N. Seiberg, *Comments on string theory on AdS_3*, *Adv. Theor. Math. Phys.* **2** (1998) 733 [hep-th/9806194].

[4] J. de Boer, H. Ooguri, H. Robins and J. Tannenhauser, *String theory on AdS_3*, *J. High Energy Phys.* **12** (1998) 026 [hep-th/9812046].

[5] D. Kutasov and N. Seiberg, *More comments on string theory on AdS_3*, *J. High Energy Phys.* **04** (1999) 008 [hep-th/9903219].

[6] J. Maldacena and H. Ooguri, *Strings in AdS_3 and* $\mathrm{SL}(2,\mathbb{R})$ *WZW model, 1*, hep-th/0001053.

[7] A.B. Zamolodchikov and V.A. Fateev, *Operator algebra and correlation functions in the two-dimensional Wess-Zumino* $\mathrm{SU}(2)\times\mathrm{SU}(2)$ *chiral model*, *Sov. J. Nucl. Phys.* **43** (1986) 657;
J. Balog, L. O'Raifeartaigh, P. Forgacs and A. Wipf, *Consistency of string propagation on curved space-times: an* $\mathrm{SU}(1,1)$ *based counterexample*, *Nucl. Phys.* **B 325** (1989) 225;
L.J. Dixon, M.E. Peskin and J. Lykken, *$N=2$ superconformal symmetry and* $\mathrm{SO}(2,1)$ *current algebra*, *Nucl. Phys.* **B 325** (1989) 329;
A. Alekseev and S. Shatashvili, *Path integral quantization of the coadjoint orbits of the virasoro group and 2D gravity*, *Nucl. Phys.* **B 323** (1989) 719;
N. Mohameddi, *On the unitarity of string propagation on* $\mathrm{SU}(1,1)$, *Int. J. Mod. Phys.* **A 5** (1990) 3201
P.M.S. Petropoulos, *Comments on* $\mathrm{SU}(1,1)$ *string theory*, *Phys. Lett.* **B 236** (1990) 151;
M. Henningson and S. Hwang, *The unitarity of* $\mathrm{SU}(1,1)$ *fermionic strings*, *Phys. Lett.* **B 258** (1991) 341;
M. Henningson, S. Hwang, P. Roberts and B. Sundborg, *Modular invariance of* $\mathrm{SU}(1,1)$ *strings*, *Phys. Lett.* **B 267** (1991) 350;

JHEP04(2000)010

S. Hwang, *Cosets as gauge slices in* SU(1,1) *strings*, *Phys. Lett.* **B 276** (1992) 451 [hep-th/9110039];
I. Bars and D. Nemeschansky, *String propagation in backgrounds with curved space-time*, *Nucl. Phys.* **B 348** (1991) 89;
S. Hwang, *No ghost theorem for* SU(1,1) *string theories*, *Nucl. Phys.* **B 354** (1991) 100;
K. Gawedzki, *Noncompact WZW conformal field theories*, hep-th/9110076;
I. Bars, *Ghost-free spectrum of a quantum string in* $\mathrm{SL}(2,\mathbb{R})$ *curved space-time*, *Phys. Rev.* **D 53** (1996) 3308 [hep-th/9503205]; *Solution of the* $\mathrm{SL}(2,\mathbb{R})$ *string in curved spacetime*, in *Future perspectives in string theory* Los Angeles, 1995 [hep-th/9511187];
O. Andreev, *On 2D gravity coupled to* $c \leq 1$ *matter in Polyakov light-cone gauge*, *Phys. Lett.* **B 375** (1996) 60 [hep-th/9601026]; *On affine Lie superalgebras,* AdS_3/CFT *correspondence and world sheets for world sheets*, *Nucl. Phys.* **B 552** (1999) 169 [hep-th/9901118], *Unitary representations of some infinite dimensional Lie algebras motivated by string theory on* AdS_3, *Nucl. Phys.* **B 561** (1999) 413 [hep-th/9905002] *Probing AdS/CFT correspondence via world-sheet methods and 2D gravity like scaling arguments*, hep-th/9909222
J.L. Petersen, J. Rasmussen and M. Yu, *Fusion, crossing and monodromy in conformal field theory based on* SL(2) *current algebra with fractional level*, *Nucl. Phys.* **B 481** (1996) 577 [hep-th/9607129];
Y. Satoh, *Ghost-free and modular invariant spectra of a string in* $\mathrm{SL}(2,\mathbb{R})$ *and three-dimensional black hole geometry*, *Nucl. Phys.* **B 513** (1998) 213 [hep-th/9705208];
J. Teschner, *On structure constants and fusion rules in the* $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)$ *WZNW model*, *Nucl. Phys.* **B 546** (1999) 390 [hep-th/9712256];
I. Pesando, *The gs type IIB superstring action on* $AdS_3 \times S^3 \times T^4$, *J. High Energy Phys.* **02** (1999) 007 [hep-th/9809145];
On the quantization of the GS type-IIB superstring action on $AdS_3 \times S^3$ *with NSNS flux*, *Mod. Phys. Lett.* **A 14** (1999) 2561 [hep-th/9903086]
J. Rahmfeld and A. Rajaraman, *The gs string action on* $AdS_3 \times S^3$ *with Ramond-Ramond charge*, *Phys. Rev.* **D 60** (1999) 064014 [hep-th/9809164];
S. Mukherji and S. Panda, *Boundary fluctuations of AdS string*, *Phys. Lett.* **B 451** (1999) 53 [hep-th/9810140];
K. Ito, *Extended superconformal algebras on* AdS_3, *Phys. Lett.* **B 449** (1999) 48 [hep-th/9811002]; *Green-schwarz superstrings on* AdS_3 *and the boundary* $N = 4$ *superconformal algebra*, *Mod. Phys. Lett.* **A 14** (1999) 2379 [hep-th/9910047];
K. Hosomichi and Y. Sugawara, *Hilbert space of space-time SCFT in* AdS_3 *superstring and* T^{4kp}/S_{kp} *SCFT*, *J. High Energy Phys.* **01** (1999) 013 [hep-th/9812100]; *Multi-strings on* $AdS_3 \times S^3$ *from matrix string theory*, *J. High Energy Phys.* **07** (1999) 027 [hep-th/9905004];
M. Yu and B. Zhang, *Light-cone gauge quantization of string theories on* AdS_3 *space*, *Nucl. Phys.* **B 551** (1999) 425 [hep-th/9812216];
N. Berkovits, C. Vafa and E. Witten, *Conformal field theory of AdS background with Ramond-Ramond flux*, *J. High Energy Phys.* **03** (1999) 018 [hep-th/9902098];
J. de Boer, A. Pasquinucci and K. Skenderis, *AdS/CFT dualities involving large 2D* $N = 4$ *superconformal symmetry*, hep-th/9904073;

M. Banados and A. Ritz, *A note on classical string dynamics on AdS_3*, *Phys. Rev.* **D 60** (1999) 126004 [hep-th/9906191];
I. Bars, C. Deliduman and D. Minic, *String theory on AdS_3 revisited*, hep-th/9907087;
P.M. Petropoulos, *String theory on AdS_3: some open questions*, hep-th/9908189;
Y. Sugawara, *Topological string on $AdS_3 \times N$*, hep-th/9909146;
G. Giribet and C. Nunez, *Interacting strings on AdS_3*, *J. High Energy Phys.* **11** (1999) 031 [hep-th/9909149];
A. Kato and Y. Satoh, *Modular invariance of string theory on AdS_3*, hep-th/0001063;
M. Langham, *Vertex operators in $AdS_3 \times S^3$ background with NS-NS flux using Berkovits-Vafa-Witten variables*, hep-th/0002032.

[8] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[9] S. Elitzur, O. Feinerman, A. Giveon and D. Tsabar, *String theory on $AdS_3 \times S^3 \times S_3 \times S^1$*, *Phys. Lett.* **B 449** (1999) 180 [hep-th/9811245].

[10] D. Kutasov, F. Larsen and R.G. Leigh, *String theory in magnetic monopole backgrounds*, *Nucl. Phys.* **B 550** (1999) 183 [hep-th/9812027].

[11] S. Yamaguchi, Y. Ishimoto and K. Sugiyama, *AdS_3/CFT_2 correspondence and space-time $N = 3$ superconformal algebra*, *J. High Energy Phys.* **02** (1999) 026 [hep-th/9902079].

[12] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080].

[13] L. Castellani et al., *G/H M-branes and AdS_{p+2} geometries*, *Nucl. Phys.* **B 527** (1998) 142 [hep-th/9803039].

[14] B.S. Acharya, J.M. Figueroa-O'Farrill, C.M. Hull and B. Spence, *Branes at conical singularities and holography*, *Adv. Theor. Math. Phys.* **2** (1999) 1249 [hep-th/9808014];
J.M. Figueroa-O'Farrill, *Near-horizon geometries of supersymmetric branes*, hep-th/9807149; *On the supersymmetries of anti de Sitter vacua*, *Class. and Quant. Grav.* **16** (1999) 2043 [hep-th/9902066].

[15] A. Giveon and M. Rocek, *Supersymmetric string vacua on $AdS_3 \times N$*, *J. High Energy Phys.* **04** (1999) 019 [hep-th/9904024];
D. Berenstein and R.G. Leigh, *Spacetime supersymmetry in AdS_3 backgrounds*, *Phys. Lett.* **B 458** (1999) 297 [hep-th/9904040].

[16] Y. Kazama and H. Suzuki, *New $N = 2$ superconformal field theories and superstring compactification*, *Nucl. Phys.* **B 321** (1989) 232;
Characterization of $N = 2$ superconformal models generated by coset space method, *Phys. Lett.* **B 216** (1989) 112.

[17] A. Schwimmer and N. Seiberg, *Comments on the $N = 2$, $N = 3$, $N = 4$ superconformal algebras in two-dimensions*, *Phys. Lett.* **B 184** (1987) 191.

[18] N. Seiberg and E. Witten, *The D1/D5 system and singular CFT*, *J. High Energy Phys.* **04** (1999) 017 [hep-th/9903224].

[19] D. Friedan, E. Martinec and S. Shenker, *Conformal invariance, supersymmetry and string theory*, *Nucl. Phys.* **B 271** (1986) 93.

[20] T. Banks, L.J. Dixon, D. Friedan and E. Martinec, *Phenomenology and conformal field theory or can string theory predict the weak mixing angle?*, *Nucl. Phys.* **B 299** (1988) 613;
T. Banks and L.J. Dixon,*Constraints on string vacua with space-time supersymmetry*, *Nucl. Phys.* **B 307** (1988) 93.

[21] T. Friedrich and I. Kath, *Seven-dimensional compact Riemannian manifolds with killing spinors*, *Comm. Math. Phys.* **133** (1990) 543.

[22] L. Castellani, L.J. Romans and N.P. Warner, *Symmetries of coset spaces and Kaluza-Klein supergravity*, *Ann. Phys. (NY)* **157** (1984) 394;
A classification of compactifying solutions for $D = 11$ supergravity, *Nucl. Phys.* **B 241** (1984) 429.

[23] P.M. Cowdall and P.K. Townsend, *Gauged supergravity vacua from intersecting branes*, *Phys. Lett.* **B 429** (1998) 281 [hep-th/9801165].

[24] H.J. Boonstra, B. Peeters and K. Skenderis, *Brane intersections, anti-de sitter spacetimes and dual superconformal theories*, *Nucl. Phys.* **B 533** (1998) 127 [hep-th/9803231].

[25] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos and P.K. Townsend, *Hyper-Kähler manifolds and multiply intersecting branes*, *Nucl. Phys.* **B 500** (1997) 133 [hep-th/9702202].

[26] G. Papadopoulos and A. Teschendorff, *Grassmannians, calibrations and five-brane intersections*, hep-th/9811034.

[27] G. Papadopoulos, *Rotating rotated branes*, *J. High Energy Phys.* **04** (1999) 014 [hep-th/9902166].

[28] A. Giveon, D. Kutasov and O. Pelc, *Holography for non-critical superstrings*, *J. High Energy Phys.* **10** (1999) 035 [hep-th/9907178].